Problem set 1

November 9, 2015

Problem 1. Do the following sets with operations form a group?

- the set of even integers $2\mathbb{Z}$ under multiplication $\cdot$? under addition "+"?
- positive reals $\mathbb{R}_{>0}$ with the usual multiplication "$\cdot$"?
- $n \times n$ matrices $Mat_{n\times n}(\mathbb{R})$ under addition of matrices "+"? under multiplication of matrices "$\cdot$"?

Problem 2. If G is a group and $g * h = h$ for some $g, h \in G$, then prove that g is necessarily the identity element e.

Problem 3. Let G be a group with 4 elements. Prove that it is necessarily abelian.

Problem 4. If G is a finite group, prove that for every $g \in G$ there is $n \in \mathbb{N}$ such that $g^n = e$. (Here g^n denotes the product $g * g * \cdots * g$ of g with itself n times).

This number n is called the **order** of the element g.

Problem 5. Show that if G is a finite group of order N, and $g \in G$ has order n, then n must divide N.

Problem 6. Describe all the groups of order p, where p is a prime number.